MTH 111 **Test 5**
Spring 2019

Calculus I

Name ______________________

Justify all answers by showing your work or by providing a coherent explanation. Please circle your answers.

1. Find the area under the curve $y = \dfrac{3}{x} + 2$ from $x = 1$ to $x = 2$.

$$\int_1^2 \frac{3}{x} + 2\, dx = 2x + 3\ln x \Big|_1^2 = 2 + 3\ln 2$$

2. A culture of bacteria is growing at a rate of $\dfrac{dy}{dt} = 20e^{0.8t}$ cells per day where t is the number of days since the culture was started. Suppose that the culture began with 40 cells. Find the formula for the total number of cells in the culture .

$$y(t) = \int 20e^{0.8t}\, dt = 25e^{0.8t} + C$$

$$y(0) = 40 \quad \Rightarrow \quad y(t) = 25e^{0.8t} + 15$$

3. Solve the following differential equation for y : $6\dfrac{dy}{dx} - x^2 = 4\sin x$.

$$\frac{dy}{dx} = \frac{1}{6}\left(4\sin x + x^2\right)$$

$$y(x) = \int \frac{2}{3}\sin x + \frac{1}{6}x^2 dx$$

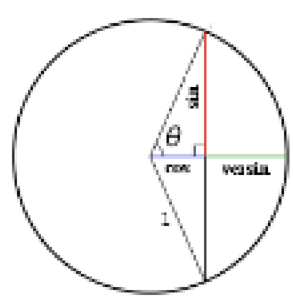
$$y(x) = -\frac{2}{3}\cos x + \frac{1}{18}x^3 + C$$

4. The *versine* or *versed sine, versin*(θ) , is a trigonometric function equal to $1 - \cos\theta$. The *haversine*, half the versine, was important in navigation because it was used to reasonably accurately compute distances on a sphere given angular positions (e.g., longitude and latitude). It is now little used.

Find $\displaystyle\int 5\, haversin(2\theta)\, d\theta$.

$$5\int \frac{1}{2}(1 - \cos 2\theta)\, d\theta = \frac{5}{2}\int (1 - \cos 2\theta)\, d\theta =$$

$$\frac{5}{2}\left(\theta - \frac{5}{2}\sin 2\theta\right) + C = \frac{5}{2}\theta - \frac{5}{4}\sin 2\theta + C$$



5. $\int (\frac{\pi}{2}x^4 - 5x^2 + ex\,)dx =$

$$\frac{1}{10}\pi x^5 - \frac{5}{3}x^3 + \frac{1}{2}ex^2 + C$$

6. $\int \left(\frac{1}{e^x} + \frac{1}{x^2} + \frac{1}{x}\right)dx =$

$$-\frac{1}{e^x} - \frac{1}{x} + \ln x + C$$

7. $\int \frac{1+\cos^2 x}{\csc x}\,dx = \int \frac{1}{\csc x} + \frac{\cos^2 x}{\csc x}dx =$

$$\int \sin x + \sin x\left(\cos^2 x\right)dx = -\cos x - \frac{1}{3}\cos^3 x + C$$

8. $\int \frac{dx}{\sin^2 x} = \int \csc^2 x\ dx = -\cot x + C$

9. $\int \tan^3 x \sec^2 x\,dx = \quad \frac{1}{4}\tan^4 x + C$

10. $\int_1^4 \frac{8}{\sqrt{x}}\,dx =$

$$8\int_1^4 x^{-\frac{1}{2}} = 16x^{\frac{1}{2}}\Big|_1^4 = 16$$

11. The voltage drop, $v(t)$, on an *RLC* circuit is given by $v(t) = R\cdot i(t) + L\cdot\frac{di}{dt} + \frac{1}{C}\int_0^t i(t)\,dt$, where R is the resistance (in ohms), L is the inductance (in henrys), and C is the capacitance (in farads). If we are given $i(t) = \sin 3t + \cos t$ (in amperes) , calculate the voltage drop after π sec with $R = 10\ \Omega$, $L = 2\ H$, and $C = 10\ F$.

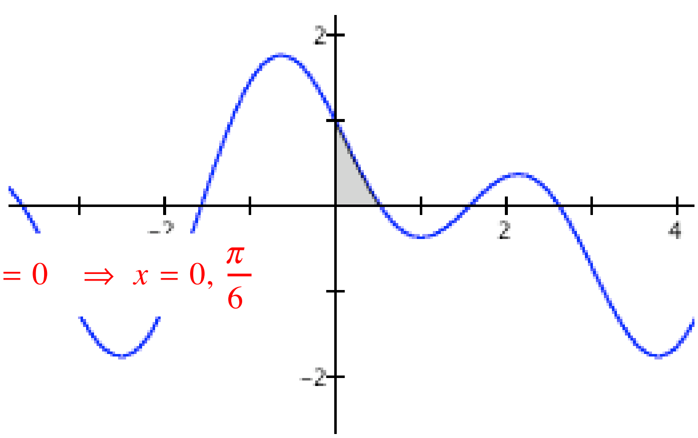
$$v(t) = 10\left(\sin 3t + \cos t\right) + 2\left(3\cos 3t - \sin t\right) + \frac{1}{10}\int_0^t \left(\sin 3t + \cos t\right)dt$$

$$= 10\left(\sin 3t + \cos t\right) + 2\left(3\cos 3t - \sin t\right) + \frac{1}{10}\left(-\frac{1}{3}\cos 3t + \sin t\right)$$

$$v(\pi) = 10\left(\sin 3\pi + \cos\pi\right) + 2\left(3\cos 3\pi - \sin\pi\right) + \frac{1}{10}\left(-\frac{1}{3}\cos 3\pi + \sin\pi\right) = -\frac{239}{15}$$

12. Find the shaded area under the curve
$y = \cos x - \sin 2x$.

(Remember: $\sin 2x = 2\sin x\cos x$)



$$\cos x - 2\sin x\cos x = 0 \quad\Rightarrow\quad \cos x\left(1 - 2\sin x\right) = 0 \quad\Rightarrow\quad x = 0,\ \frac{\pi}{6}$$

$$\int_0^{\frac{\pi}{6}} \cos x - \sin 2x\ dx = \sin x + \frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{6}} = \frac{1}{4}$$